Fixed Effects Estimation

- When there is an observed fixed effect, an alternative to first differences is fixed effects estimation
- Consider the average over time of $y_{it} = \beta_1 x_{it1} + \ldots + \beta_k x_{itk} + a_i + u_{it}$
- The average of a_i will be a_i, so if you subtract the mean, a_i will be differenced out just as when doing first differences

Fixed Effects Estimation (cont)

- If we were to do this estimation by hand, we'd need to be careful because we'd think that $df = NT - k$, but really is $N(T - 1) - k$ because we used up *df*s calculating means
- Luckily, Stata (and most other packages) will do fixed effects estimation for you
- This method is also identical to including a separate intercept for every individual

First Differences vs Fixed Effects

- First Differences and Fixed Effects will be exactly the same when T = 2
- For T > 2, the two methods are different
- Probably see fixed effects estimation more often than differences – probably more because it's easier than that it's better
- Fixed effects easily implemented for unbalanced panels, not just balanced panels

Random Effects

- Start with the same basic model with a composite error, $y_{it} = \beta_0 + \beta_1 x_{it1} + \ldots \beta_k x_{itk} + a_i + u_{it}$
- Previously we've assumed that a_i was correlated with the x's, but what if it's not?
- OLS would be consistent in that case, but composite error will be serially correlated

Random Effects (continued)

- Need to transform the model and do GLS to solve the problem and make correct inferences
- Idea is to do quasi-differencing with the

Random Effects (continued)

- Need to transform the model and do GLS to solve the problem and make correct inferences
- End up with a sort of weighted average of OLS and Fixed Effects – use quasi-demeaned data

$$\lambda = 1 - \left[\sigma_u^2 / \left(\sigma_u^2 + T\sigma_a^2\right)\right]^{1/2}$$

$$y_{it} - \lambda\bar{y}_i = \beta_0\left(1-\lambda\right) + \beta_1\left(x_{it1} - \lambda\bar{x}_{i1}\right) + \ldots$$

$$+ \beta_k\left(x_{itk} - \bar{x}_{ik}\right) + \left(\nu_{it} - \bar{\nu}_i\right)$$

Random Effects (continued)

- If $\lambda = 1$, then this is just the fixed effects estimator
- If $\lambda = 0$, then this is just the OLS estimator
- So, the bigger the variance of the unobserved effect, the closer it is to FE
- The smaller the variance of the unobserved effect, the closer it is to OLS
- Stata will do Random Effects for us

Fixed Effects or Random?

- More usual to think need fixed effects, since think the problem is that something unobserved is correlated with the *x*'s
- If truly need random effects, the only problem is the standard errors
- Can just adjust the standard errors for correlation within group

Other Uses of Panel Methods

- It's possible to think of models where there is an unobserved fixed effect, even if we do not have true panel data
- A common example is where we think there is an unobserved family effect
- Can difference siblings
- Can estimate family fixed effect model

Additional Issues

- Many of the things we already know about both cross section and time series data can be applied with panel data
- Can test and correct for serial correlation in the errors
- Can test and correct for heteroskedasticity
- Can estimate standard errors robust to both